P: ISSN No. 2231-0045 RNI No. UPBIL/2012/55438 VOL.-7, ISSUE-4, May-2019

E: ISSN No. 2349-9435 Periodic Research

MHD Flow and Heat Transfer along Vertical Permeable Stretching/Shrinking Sheet with Second Order Slip

Abstract

In the present paper, MHD flow and heat transfer over vertical permeable stretching/shrinking sheet with second order slip is analyzed. The governing non-linear partial differential equations are transformed into ordinary differential equationsby similarity transformation and solved numerically using Runge-Kutta fourth order method with shooting technique. Effects of various physical quantities such as magnetic parameter, mixed convection parameter, suction parameter, first order slip, second order slip and Prandtl number on flowfield and heat transfer characteristic are presented through graphsand discussed numerically.

Keywords: MHD**,** Vertical Sheet, Stretching, Shrinking, Second Order Slip.

Deepak Kumar
Research Scholar,
Deptt. of Mathematics,
University of Rajasthan,
Jaipur

Sharad Sinha
Assistant Professor,
Deptt. of Mathematics,
University of Rajasthan,
Jaipur

Anil Sharma
Associate Professor
Deptt. of Mathematics,
University of Rajasthan,
Jaipur

Introduction

The flow and heat transfer due to stretching/shrinking surface has gained much attention due to its various applications in engineering such as extrusion processes, drawing of plastic films and wires etc., see Tadmor and Klein(1970). Work of Sakiadis (1961) and Crane (1970) are fundamental in this direction. Wang (1984) obtained analytic solution of three dimensional flow problem due to stretching flat surface. Viscous flow due to a shrinking sheet was considered by Miklavcicand Wang (2006).

Review of Literature

Suction/injection plays significant role in boundary layers control. Also, it has a notable effect on rate of heat and mass transfer. Effect of suction/ blowing on heat and mass transfer on stretching surface was considered by Gupta and Gupta (1977), Chen and Char (1988). Fang and Zhang (2009) found closed form solution of MHD viscous flow over shrinking sheet. It is the fact that in viscous fluid flow, the molecules normally stick to the surface but when length scale is less than the mean free path of the molecules, the molecules slip at the wall and the no-slip boundary condition is no longer valid.

Widely used slip model is 1^{st} order slip model also known as Maxwell slip model. Andersson and Wang (2002) considered first order slip flow past a stretching surface. Fang, Zhang and Yao (2010) analyzed slip magnetohydrodynamic viscous flow problem with suction/blowing over a shrinking sheet and obtained analytical solution. For high value of Knudsen number, it becomes difficult to predict 1^{st} order slip model. Wu(2008) gave a slip model valid for arbitrary Knudsen number. Later Fang and Aziz(2010), Fang et al. (2010) used this model in flow over stretching/shrinking sheet with suction/blowing.

Nandeppanavar et al. (2012) studied second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition. Singh and Chamkha(2013) investigated dual solutions for second order slip flow on a vertical permeable shrinking sheet. Heat and mass transfer of MHD second order slip flow was discussed by Turkyilmazoglu(2013). Effects of second order slip on the flow of a fractional Maxwell MHD fluid was analyzed by Liu and Guo (2017) , also Majid (2018) analyzed activation energy with the second order slip condition.

In all the studies mentioned above, second order slip model was taken with stretching/shrinking sheet in absence of external magnetic field and when the magnetic field considered, the study was done only for forced convection.

Aim of the Study

The objective of the present paper is to study MHD mixed convective flow and heat transfer along vertical permeable stretching sheet with second order slip.

Formulation of the Problem

Consider steady, two-dimensional laminar boundary layer slip flow of viscous incompressible electrically conducting fluidalong avertical permeable linearly stretching/shrinking sheet. The sheet is placed along x-axis and y-axis is normal to it. A uniform external magnetic field of strength B_o is applied in the direction normal to the sheet. The linearly stretching/shrinking velocity and temperature of the sheet are $U_w(x) = dcx$ and $T_w(x)$ respecitvely. Under these conditions along with the Boussinesq approximation, equations governing the problem are given by

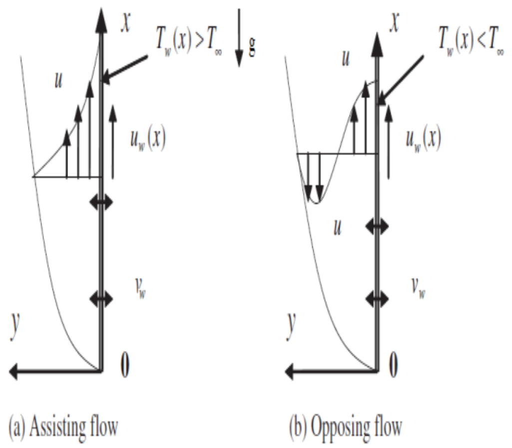

Fig1: Physical model of the problem and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_\infty), \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \quad (3)$$

subject to the boundary conditions

$$\left.\begin{array}{l} u = U_w(x) + u_{slip}\ ,\ v = v_w,\ \ T = T_w = T_\infty + T_o x\ \ at\ \ y = 0; \\ u \to 0,\ \ T \to T_\infty\ \ as\ \ y \to \infty; \end{array}\right\} \quad (4)$$

where u and v are the velocity components along x and y directions respectively, g is the acceleration due to gravity, $\nu(=\frac{\mu}{\rho})$ is the kinematic viscosity, μ is the coefficient of viscosity, ρ is the density of the fluid, σ is the electrical conductivity, β is the coefficient of the thermal expansion, T is the fluid temperature, κ is the thermal conductivity, C_p is the specific heat at constants pressure, T_∞ is the ambient temperature. Further d and c are constants, where c is positive and d is 1 for stretching and -1 for shrinking sheet. $v_w$is the suction/injection velocity, $v_w < 0$ for suction and $v_w > 0$ for injection, respectively. $T_o$is the characteristic temperature with $T_o > 0$ for heated sheet(assisting flow) and $T_o < 0$ for cooled sheet (opposing flow).

$u_{slip}$is the slip velocity (Wu (2008) and Fang et al.(2010)) at the sheet, given by

$$\begin{aligned} u_{slip} &= \frac{2}{3}\left(\frac{3 - \alpha l^3}{\alpha} - \frac{3}{2}\frac{1 - l^2}{K_n}\right)\varepsilon\frac{\partial u}{\partial y} \\ &\quad - \frac{1}{4}\left(l^4 + \frac{2}{K_n^2}(1 - l^2)\right)\varepsilon^2\frac{\partial^2 u}{\partial y^2} \\ &= A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} \end{aligned}$$

where $l = min(\frac{1}{K_n}, 1)$, α $(0 \le \alpha \le 1)$ is the momentum accommodation coefficient and $\varepsilon(> 0)$ is molecular mean free path,also for any given value of Knudsen number (K_n), we have $0 \le l \le 1$ and as a result A is positive and B is negative.

Method of Solution

To convert partial derivatives occurring in equations (1)-(3) with boundary conditions (4) into ordinary derivatives, we introduce the following similarity variables

$$\eta = y\sqrt{\frac{c}{\nu}}, \psi = \sqrt{c\nu}\, x f(\eta) \text{and } \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (5)$$

Such that$u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. The equation of continuity (1) is automatically satisfied. Equations (2) and (3) are reduced to

$$f''' + ff'' - f'^2 - Mf' + \lambda\theta = 0 \quad (6)$$

$$\theta'' + \Pr(f\theta' - f'\theta) = 0 \quad (7)$$

with boundary conditions

$$\left.\begin{array}{l} f(0) = S\ ,\ f'(0) = d + \gamma f''(0) + \delta f'''(0)\ \ and\ \ \theta(0) = 1\ ; \\ f'(\eta) \to 0\ \ and\ \ \theta(\eta) \to 0\ \ as\ \eta \to \infty \end{array}\right\} \quad (8)$$

where $M\left(= \frac{\sigma\beta_0^2}{\rho c}\right)$ is magnetic parameter, $\lambda\left(= \frac{Gr_x}{Re_x^2}\right)$ is mixed convection parameter,$Gr_x\left(= \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}\right)$ is the local Grashof number,$Re_x\left(= \frac{U_w(x)x}{\vartheta}\right)$ is the local Reynolds number, $Pr\left(= \frac{\nu\rho C_p}{\kappa}\right)$ is the Prandtl number, $S\left(= -\frac{v_w}{\sqrt{c\vartheta}}\right)$ is suction/injection parameter ($S > 0$ for suction and $S < 0$ for injection), $\gamma\left(= A\sqrt{\frac{c}{\vartheta}}\right)$ is the first order slip parameter and $\delta\left(= B\frac{c}{\vartheta}\right)$ is the second order slip parameter. In this paper only suction is considered which helps retaining the vorticity within the boundary layer and maintaining the laminar flow.

Physical quantities of interest are Skin friction coefficient (C_f) and Nusselt number (Nu_x)

$$C_f = \frac{\tau_w}{\rho U_w^2}\ , \qquad Nu_x = \frac{x q_w}{\kappa(T_w - T_\infty)} \quad (9)$$

where the wall shear stress τ_w and the heat flux at the wall q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using similarity transformation (5) in (9), the skin friction coefficient and local Nusselt number reduced to

$$C_f Re_x^{1/2} = f''(0)\ ,\ Nu_x Re_x^{-1/2} = -\theta'(0).$$

Since equations (6) and (7) are highly nonlinear, these equations with boundary conditions (8) are converted into system of first order differential equations as given in (10)

$$f_1' = f_2, f_2' = f_3\ , f_3' = -f_1 f_3 + M f_2 + {f_2}^2 - \lambda f_4\ , f_4' = f_5\ , f_5' = \Pr(f_4 f_2 - f_1 f_5), \quad (10)$$

and boundary conditions are

$$\left.\begin{matrix} f_1(0) = S,\ f_2(0) = 1 + \gamma f_3(0) + \delta f_3'(0), f_4(0) = 1; \\ \text{and} f_2 \to 0,\ \ f_4 \to 0 as \eta \to \infty \end{matrix}\right\} \quad (11)$$

Where $f_1 = f$ and $f_4 = \theta$.

System of first order differential equations given in (10) is solved numerically by Runga-Kutta(fourth order) method, but in (11) only three initial conditions are given. To overcome this problem shooting technique is used, the initial guesses for $f_3(0)$ and $f_5(0)$ are assumed to be $s_1$and s_2. In order to validate the assumed values of $s_1$and s_2, calculated values of $f_2$and f_4 at the upper boundary ($\eta \to \infty$) are compared with the values given in (11). The procedure is repeated until the results upto the desired degree of accuracy are obtained.

Results and discussion

Effects of various physical parameters on fluid velocity, fluid temperature, skin friction coefficient and Nusselt number are discussed in this section. Impact of these parameters for stretching sheet are presented through figure (2) to (7) and table1, while for shrinking sheet it is given by figures (8), (9) and table 2.

Figure 2(a) and 2(b) represent that fluid velocity decreases and temperature increases with increasing magnetic parameter, as increasing magnetic parameter causes more Lorentz force which work against the motion of the fluid and retards it. Due to this both skin friction and Nusselt number decrease which can be verified from table1.

If mixed convection parameter increases, fluid velocity increases while fluid temperature decreases as shown in figure 3(a) and 3(b). This is because, buoyancy effect enhances with increasing mixed convection parameter.

Figure 4(a) and 4(b) represent that both fluid velocity and temperature decreases with increasing values of suction parameter. Some amount of fluid sucked by the wall due to suction parameter and when it is large more amount of fluid will be sucked and as result skin friction decreases and Nusselt number increases.

Effect of first order slip parameter on fluid velocity and temperature are shown through figures 5(a) and 5(b), from figure it is clear that fluid velocity decreases while temperature increases with enhancing first order slip parameter which result in thinning velocity boundary layer and thickening thermal boundary layer.

There is similar effect of absolute value of second order slip parameter $|\delta|$on fluid velocity and temperature as shown in figure 6(a) and 6(b). It is to be noted that variation in temperature profile due to change in both first and second order velocity slip parameters is very small.

Figure 7 depicts the effect of Prandtl number on fluid temperature. When Prandtl number increases, kinematic viscosity dominates thermal diffusivity and as a result temperature of the fluid decreases and Nusselt number increases.

Figures 8 and 9 illustrate the effect of first and second order velocity slip parameters on fluid velocity and fluid temperature for shrinking sheets. From the figures 8(a) and 9(a) it is clear that velocity boundary layer thickness decreases with increasing values of first order slip parameter $\gamma$and absolute values of second order slip parameter$|\delta|$.Effect of these two parameters on fluid temperature is shown through figures 8(b) and 9(b). There is very low impact of velocity slip parameters on temperature profile.As first order slip parameter γ or absolute values of second order slip parameter i.e. $|\delta|$ increases fluid temperature decreases. The effect of these parameter on fluid temperature for shrinking sheet are opposite to that of stretching sheet as seen in figure 5(b) and 6(b).

Influence of these physical parameters on skin friction coefficient and Nusselt number are shown through table 1. From the numerical values given in the table it is clear that both skin friction coefficient and Nusselt number decreases with increasing values of magnetic parameter, while opposite impact is noticed for mixed convection parameter. When suction parameter increases skin friction coefficient decrease and Nusselt number increase. Nusselt number increases rapidly with increasing Prandtl number. As first order velocity slip parameter or absolute value of second order slip parameter increases both skin friction coefficient and Nusselt number decrease for stretching sheet while these parameters have opposite impact on Nusselt number for shrinking sheet.

Conclusions

A numerical study has been carried out on MHD flow and heat transfer along vertical permeable stretching/shrinking sheet in the presence of first and second order velocity slips. Some important findings from the present analysis are as follows:

1. Increasing values of first order slip parameter and absolute values of second order slip parameter reduce the shear stress at the wall.
2. Effect of first order velocity parameter and absolute value of second order slip parameter on Nusselt number are reverse for stretching and shrinking sheets respectively.
3. Combination of mixed convection parameter and Prandtl number greatly affect the temperature gradient at the wall as well as in the fluid.

Fig2a: Effect of Magnetic parameter (M) onvelocity $f'(\eta)$ with $\eta$for stretching sheet when $= 0.1, s = 2.0,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

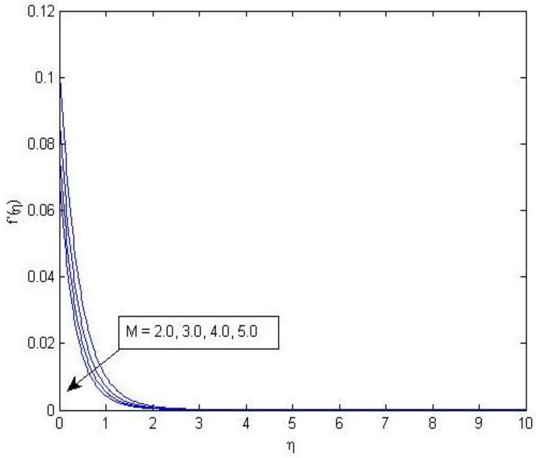

Fig2b: Effect of Magnetic parameter(M) on temperature $\theta(\eta)$ with η for stretching sheet when $\lambda = 0.1, s = 2.0,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

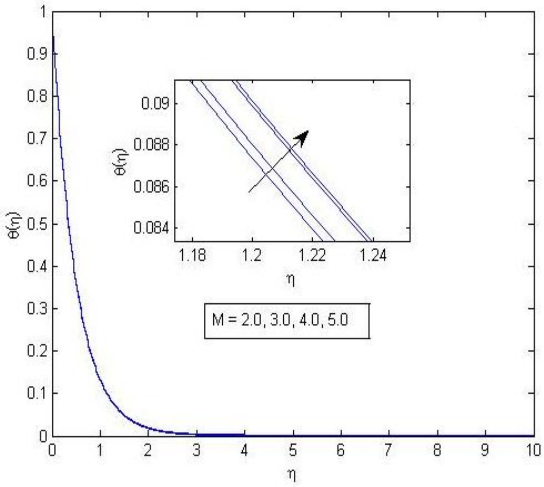

Fig3a: Effect of Mixed convection parameter (λ) on velocity $f'(\eta)$ with η for stretching sheet when $M = 2.0, s = 2.0,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

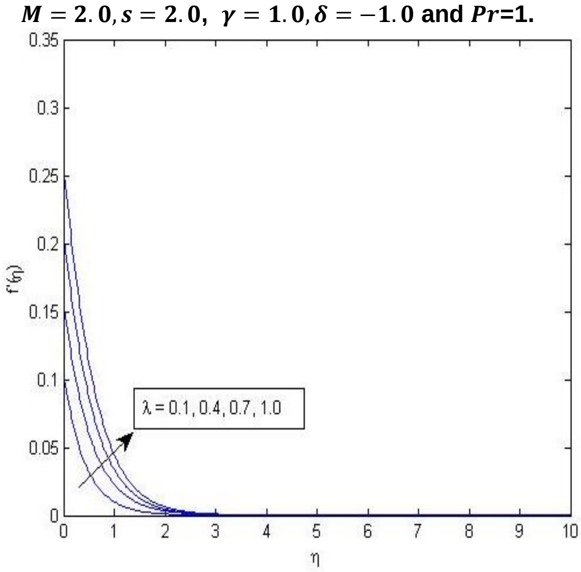

Fig3b: Effect of Mixed convection parameter (λ) on temperature $\theta(\eta)$ with η for stretching sheet when $= 2.0, s = 2.0,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

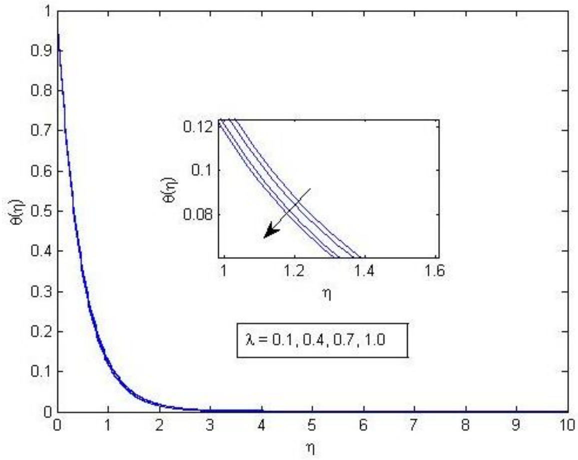

Fig4a: Effect of suction parameter(s) on velocity $f'(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

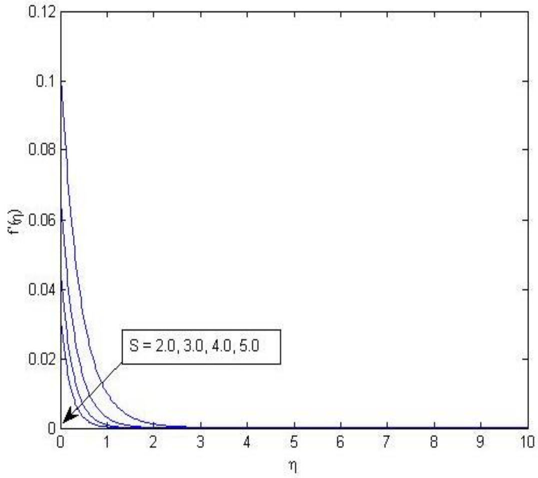

Fig4b: Effect of suction parameter (s) on temperature $\theta(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ \gamma = 1.0, \delta = -1.0$ and Pr=1.

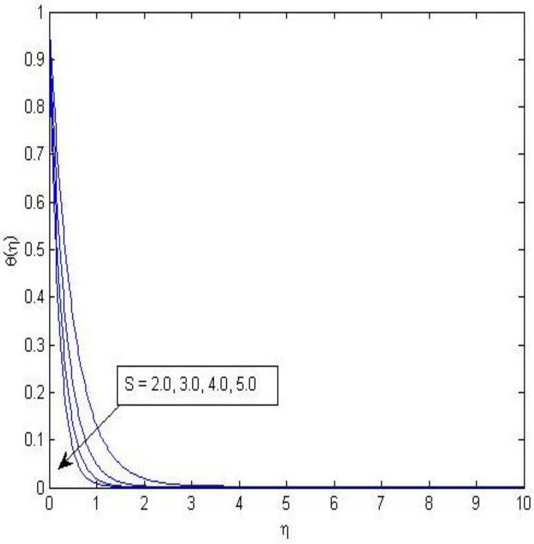

Fig5a: Effect of first order slip parameter(γ) on velocity $f'(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ S = 2.0, \delta = -1.0$ and Pr=1.

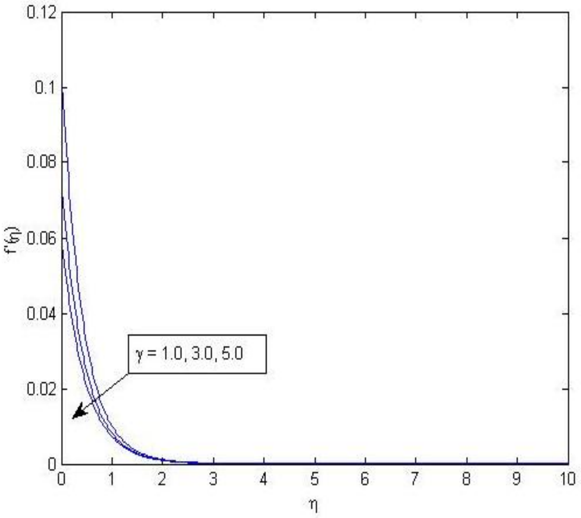

Fig5b: Effect of first order slip parameter (γ) on temperature $\theta(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ S = 2.0, \delta = -1.0$ and Pr=1.

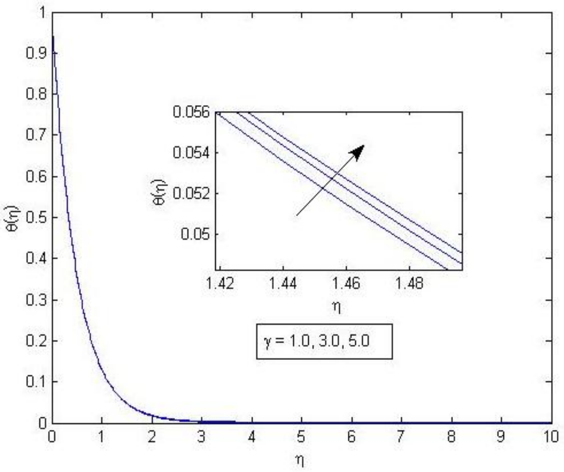

Fig6a: Effect of second order slip parameter (δ) on velocity $f'(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ s = 2.0, \gamma = 1.0$ and Pr=1.

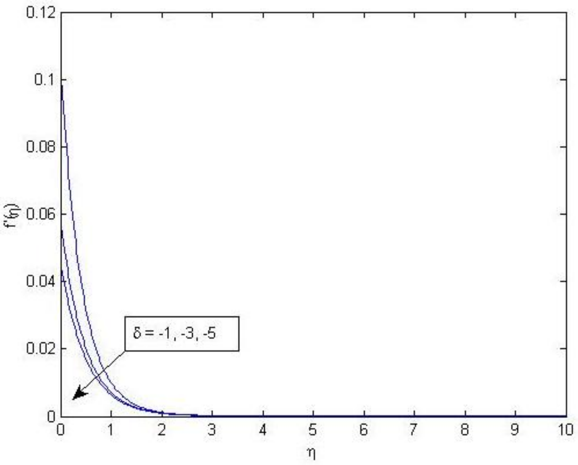

Fig6b: Effect of second order slip parameter (δ) on temperature $\theta(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ s = 2.0, \gamma = 1.0$ and Pr=1.

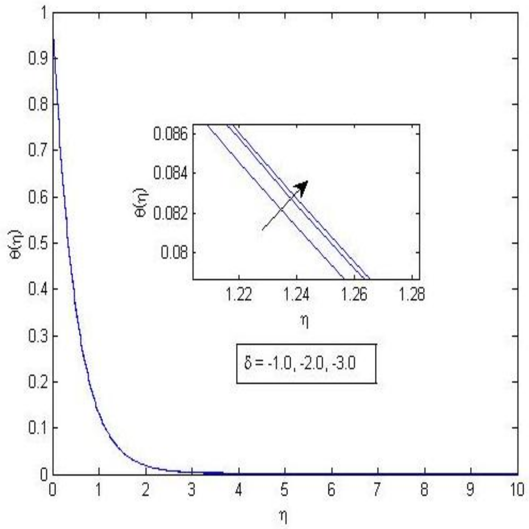

Fig7: Effect of Prandtl number (Pr) on temperature $\theta(\eta)$ with η for stretching sheet when $M = 2.0, \lambda = 0.1,\ s = 2.0, \gamma = 1.0$ and $\delta = -1.0$.

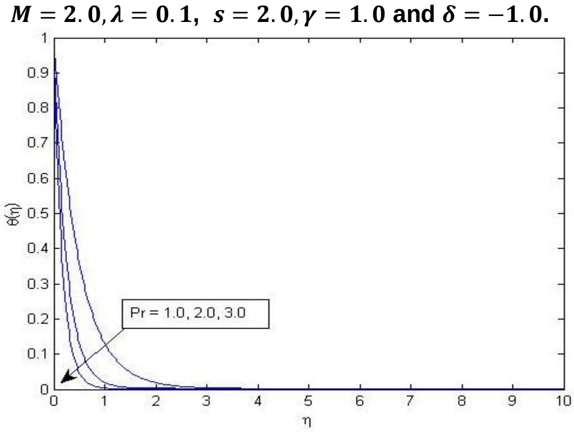

Fig 8a: Effect of first order slip parameter (γ) on velocity $f'(\eta)$ with η for shrinking sheet when $M = 2.0, \lambda = 0.1,\ S = 2.0, \delta = -1.0$ and Pr=1.

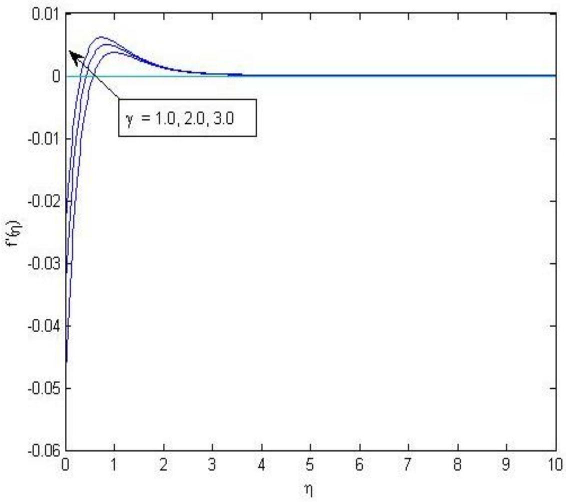

Fig 8b: Effect of first order slip parameter (γ) on temperature $\theta(\eta)$ with η for shrinking sheet when $M = 2.0, \lambda = 0.1,\ S = 2.0, \delta = -1.0$ and Pr=1.

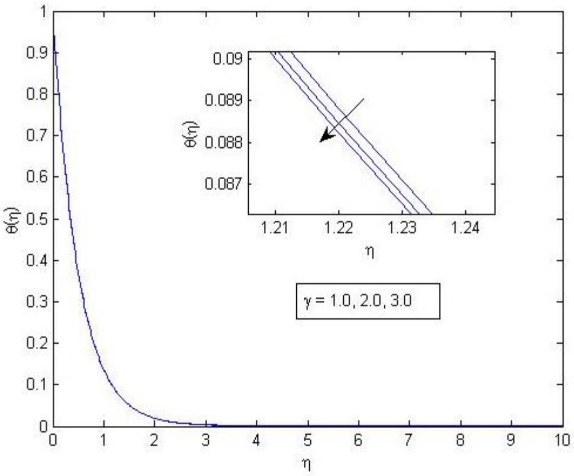

Fig 9a: Effect of second order slip parameter (δ) on velocity $f'(\eta)$ with η for shrinking sheet when $M = 2.0, \lambda = 0.1,\ s = 2.0, \gamma = 1.0$ and Pr=1.

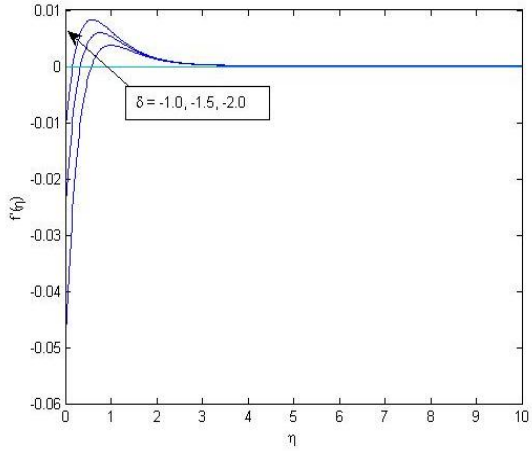

Fig 9b: Effect of second order slip parameter (δ) on temperature $\theta(\eta)$ with η for shrinking sheet when $M = 2.0, \lambda = 0.1,\ s = 2.0, \gamma = 1.0$ and Pr=1.

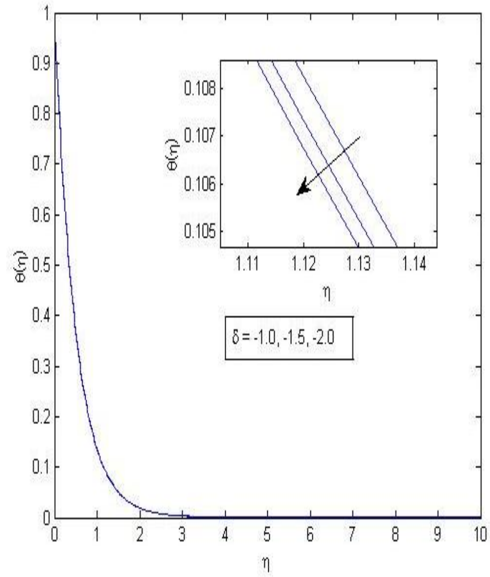

Table 1: Numerical values of skin friction coefficient and Nusselt number for different values of physical parameters when d=1 (i.e. stretching sheet).

M	λ	S	γ	δ	Pr	$f''(0)$	$-\theta'(0)$
2	0.1	2.0	1	-1	1	0.25692846	2.043
3						0.24249189	2.038
4						0.23068997	2.028
5						0.22083509	2.024
2	0.1	2.0	1	-1	1	0.25692846	2.043
	0.4					0.29963775	2.074
	0.7					0.34293875	2.108
	1.0					0.38594375	2.135
2	0.1	2.0	1	-1	1	0.25692846	2.043
		3.0				0.22151100	3.020
		4.0				0.19084110	4.010
		5.0				0.16595050	5.000
2	0.1	2.0	1	-1	1	0.25692846	2.043
			3			0.17327330	2.035
			5			0.13063525	2.028

2	0.1	2.0	1	-1	1	0.25692846	2.043
				-3		0.12543938	2.025
				-5		0.08956948	2.020
2	0.1	2.0	1	-1	1	0.25692846	2.043
					1.5	0. 25955971	3.050
					2	0.26104291	4.050
					2	0.26276848	6.060

Table 2: Numerical values of skin friction coefficient and Nusselt number for different values of physical parameters when d=-1 (i.e. shrinking sheet).

M	λ	S	γ	δ	Pr	$f''(0)$	$-\theta'(0)$
2.0	0.2	2.0	1	-1	1	0.215465000	1.98400
			2			0.172970600	1.99000
			3			0.144507700	1.99400
2.0	0.2	2.0	1	-1	1	0.215465000	1.98400
				-1.5		0.14795320	1.994
				-2.0		0.10751488	2.000

References

1. *Andersson, H.I., "Slip flow past a stretching surface". ActaMechanica, Vol. 158, 2002, pp. 121-125.*
2. *Chen, C.K.; Char, M.I., "Heat transfer of a continuous stretching surface with suction or blowing". Journal of Mathematical analysis and applications, Vol. 135, 1988, pp. 568-580.*
3. *Crane, L. J., "Flow past a stretching plate". ZeitschriftFürangewandteMathematik und Physik(ZAMP), Vol. 21(4), 1970, pp. 645–647.*
4. *Fang, T., Yao, S.S.; Zhang, J.; Aziz, A., "Viscous flow over a shrinking sheet with second order slip flow model". Communication in Nonlinear Science and Numerical Simulations. Vol. 15, 2010 pp. 1831–1842.*
5. *Fang, T.; Aziz, A., "Viscous flow with Second-order slip velocity over a stretching sheet". Z Naturforsch Vol. 65, Issue 12, 2010, pp. 1087-1092.*
6. *Fang, T.; Zhang J. "Closed form exact solutions of MHD viscous flow over shrinking sheet". Commun Nonlinear SciNumerSimulat, Vol. 14, 2009, pp. 2853-2857.*
7. *Fang, T.; Zhang, J.; Yao., S.S., " Slip magnetohydrodynamic viscous flow over a permeable shrinking sheet". Chin. Phys. Lett., Vol. 27(12), 2010, pp. 124702(1)-124702(4).*
8. *Gupta, P.S.; Gupta, A.S., "Heat and mass transfer on a stretching sheet with suction or blowing". The Canadian Journal of Chemical Engineering, Vol. 55, 1977, pp. 744-746.*
9. *Liu, Y.;Guo, B., "Effects of second-order slip on the flow of a fractional Maxwell MHD fluid". Journal of the Association of Arab Universities for Basic and AppliedSciences, Vol. 24(1), 2017, pp. 232-241.*
10. *Majeed, A.; Khan, I., "Analysis of activation energy of in magnetohydrodynamic flow with chemical reaction and second order slip model". Case Studies in Thermal Engineering, Vol. 12, 2018, pp. 765-773.*
11. *Miklavcic, M.; Wang, C.Y.,"Viscous flow due to shrinking sheet". Quarterly of applied Mathematics, Vol. 64(2), 2006, pp.283–290.*
12. *Nandeppanavar, M.M.; Vajravelu, K.; Abel, M.S.; Siddalingappa, M.N., "Second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition". International Journal of Thermal Sciences, Vol. 58, 2012, pp. 143–150.*
13. *Sakiadis, B.C., "Boundary-layer behavior on continuous solid surface: II. The boundary layer on a continuous flat surface". American Institute of Chemical Engineering Journals, Vol. 7(2), 1961, pp. 221-225.*
14. *Singh, G.; Chamkha, A.J., "Dual solutions for second-order slip flow and heat transfer on a vertical permeable shrinking sheet". Ain Shams Engineering Journal, Vol. 4, 2013, pp. 911-917.*
15. *Tadmor, Z.; Klein, I., "Engineering principles of plasticating extrusion: Polymer science and engineering series". Van Nostrand Reinhold Co., New York 1970.*
16. *Turkyilmazoglu, M., "Heat and mass transfer of MHD second order slip flow". Computers & Fluids, Vol. 71, 2013, pp. 426–434.*
17. *Wang, C.Y., "Flow due to a stretching boundary with partial slip- an exact solution of the Navier-Stokes equations". Chemical Engineering Science, Vol. 57, 2002, pp. 3745–3747.*
18. *Wang, C.Y., "The three dimensional flow due to a stretching flat surface". Physics of fluids, Vol. 27(18), 1984, pp. 1915-1917.*
19. *Wu, L., "A slip model for rarefied gas flows at arbitrary Kundsen Number". Applied Physics Letters, Vol. 93, 2008, pp. 253103(1)-253103(3).*